

Recitation 8: Doob's inequality

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Exercise 1. Suppose that M_n is a martingale and define $\Delta M_n = M_{n+1} - M_n$. Show that for every $n \neq m$,

$$\mathbb{E}[\Delta M_m \Delta M_n] = \mathbb{E}[\Delta M_m] \mathbb{E}[\Delta M_n].$$

Exercise 2. Prove that positive martingale $(X_n)_{n \in \mathbb{N}_+}$ converges almost surely to a limit.

Exercise 3 (Gambler's ruin 1). A gambler wins or loses one pound in each round of betting, with equal chances and independently of the past events. He starts betting with the determination that she will stop gambling when either he won a pounds or he lost b pounds.

1. What is the probability that she will be winning when he stops playing further ?
2. What is the expected number of his betting rounds before he will stop playing further ?

Exercise 4 (Gambler's ruin 2). Answer the same questions as in problem 2 when the probability of winning or losing one pound in each round is p , respectively, $q := 1 - p$, with $p \in (0, 1)$.